DOCUMENT RESUME

ED 253 403 SE 045 366

TITLE	Recommended Arithmetic Methods. Notes for Teachers of Mathematics, Volume 7.
INSTITUTION	Leeds Education Authority (England). Mathematics Curriculum Study Group.
PUB DATE	[81]
NOTE	16p.; For related documents, see SE 045 363-367.
AVAILABLE FROM	Leeds City Council Department of Education, John Taylor Teachers Centre, 53 Headingley Lane, Leeds LS6 1AA England (.37 pounds).
PUB TYPE	Guides - Classroom Use - Guides (For Teachers) (052)
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	*Algorithms; Decimal Fractions; *Elementary School Mathematics; Elementary Secondary Education; Foreign Countries; Fractions; *Mathematical Vocabulary; Mathematics Curriculum; *Mathematics Instruction; Measurement; Percentage; Reference Materials; *Teacher Developed Materials; *Teaching Methods
IDENTIFIERS	*England

ABSTRACT

This is one of a series of monographs developed by teachers in schools near Leeds, England. This volume focuses on recommended methods for teaching operations with whole numbers, fractions, decimals, and percentages. For children to attain confidence in dealing with arithmetical processes, it is considered important that they not be confused by changes in methods or modes of presentation which they might encounter on transferring to a different school or teacher. Thus, certain methods are recommended to encourage some measure of uniformity of practice in primary, junior, and middle schools. The sections consider the four rules applied to: whole numbers; fractions; decimals; decimal presentation of metric units; money; and conversion of decimals, fractions, and percentages. (MNS)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

NOTES FOR TEACHERS OF MATHEMATICS

ED253403

U.S. DEPARTMENT OF EDUCATION
NATIONAL INSTITUTE OF EDUCATION
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
✓ This document has been reproduced as received from the person or organization originating it.
Minor changes have been made to improve reproduction quality.
• Points of view or opinions stated in this document do not necessarily represent official NIE position or policy.

VOLUME 7

RECOMMENDED ARITHMETIC METHODS

LEEDS EDUCATION AUTHORITY

MATHEMATICS CURRICULUM STUDY GROUP

THE JOHN TAYLOR TEACHERS' CENTRE

"PERMISSION TO REPRODUCE THIS MATERIAL IN MICROFICHE ONLY HAS BEEN GRANTED BY
R.S. Johnson
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

SE 045 366

RECOMMENDED ARITHMETIC METHODS

FOREWORD

One of the aims of basic number work is to provide children with the mechanical skill necessary to deal with everyday situations in number and with the more advanced mathematics taught in secondary and high schools.

For children to attain confidence in dealing with arithmetical processes, it is important that they should not be confused by changes in methods or modes of presentation which they could encounter on transferring to a different school or teacher.

The following document is a development from papers prepared by Area Working Parties. The recommended arithmetic methods it contains have been prepared by a Working Party of the Mathematics Curriculum Study Group whose aim has been to encourage some measure of uniformity of practice in primary, junior & middle schools. Some of the suggested modes of presentation, e.g. the operator "+" sign at the bottom left in addition calculations, are partly as a result of their overwhelming common usage in contemporary published material.

Headteachers are invited to adopt the suggestions for each process outlined, but the approach to given processes will, of course, be very individual.

A/G/17

WORKING PARTY MEMBERS

Mr D. Slater	Calverley Parkside J.I. School
Miss K. Bullough	Hugh Gaitskell Middle School
Mrs E. Jackson	Christchurch Upper Armley C. of E.
Mrs E. M. Johnson	Barwick in Elmet Infants School
Mrs W. A. Poulter	Horsforth West End J.I. School
Mr R. T. Arnold	Advisory Division
Mr R. Baker	Royal Park Middle School
Mr P. Lee	Moor Grange High School
Mr J. Mason	Rothwell West Junior School
Mr A. Saxton	John Taylor Teachers' Centre

CONTENTS

		Page
THE FOUR RULES - APPLIED TO WHOLE NUMBERS		
1	Addition	1
2	Subtraction	1
3	Multiplication	2
4	Division	2
FRACTIONS		
1	Addition and subtraction of fractions having the same denominator	3
2	Addition and subtraction of fractions having different denominators - making use of equivalent fractions	4
3	Dealing with mixed numbers	4
4	Multiplication by a whole number	4
5	Multiplication by a fraction	5
6	Multiplication involving mixed numbers	5
7	Division by a whole number	5
8	Division by a fraction	5
9	Division involving mixed numbers	5
DECIMALS		
1	Addition and subtraction	6
2	Multiplication and division by powers of 10	6
3	Multiplication of two decimals	7
4	Adding '0's as required	7
5	Division	7
DECIMAL PRESENTATION OF METRIC UNITS		8
MONEY		
1	Addition	9
2	Subtraction	10
3	Multiplication	10
4	Division	10
CONVERSION OF DECIMALS, FRACTIONS AND PERCENTAGES		
1	Changing fractions to percentages	10
2	Changing decimals to percentages	10
3	Changing percentages to fractions	11
4	Changing decimals to fractions	11
5	Changing fractions to decimals	11
6	Changing percentages to decimals	11

ARITHMETIC METHODS

GENERAL NOTE

For a sound knowledge and understanding of arithmetical processes the formal (written) recording of calculations should be preceded by experience with concrete materials, e.g. Unifix material, cuisenaire rods, Tillich material.

THE FOUR RULES - APPLIED TO WHOLE NUMBERS

Note: i For addition, subtraction and multiplication it is recommended that the operator sign be placed at the 'bottom left', as indicated in the examples which follow.

ii Care should be taken at all times to ensure that figures are correctly positioned in their columns: the use of squared paper facilitates this.

1 Addition

Note: The method of presentation suggested below does not imply that children should not also be given practice in horizontal addition.

The 'carrying' figure, when used, should be placed in its appropriate column below the answer, but could ultimately be dispensed with.

$$\begin{array}{r} 123 \\ 34 \\ 250 \\ +\ 95 \\ \hline 502 \\ \hline 21 \end{array}$$

Column headings (U's, T's, H's etc.) will be used in the early stages but should be dispensed with as soon as may be possible.

2 Subtraction

Note: i The decomposition method should be used for written processes.

ii The 'complementary addition' aspect of subtraction should receive attention, e.g. "What number must be added to 5 to make 9?" "What number must be added to 37 to make 84?".

iii Practice should be given in horizontal as well as vertical subtraction, e.g.

$$\begin{array}{r} 14 \\ -\ 5 \\ \hline 9 \\ \hline \end{array} \quad \text{and } 14 - 5 = 9$$

71	291	4991
84	304	5002
-37	-176	-3657
47	128	1345

If felt desirable, the 'equal additions' method of subtraction could be used with 12+ age range pupils.

3 Multiplication

a The 'carrying' figures should be placed in the appropriate column below the answer but could ultimately be dispensed with.

```
  57
x  6
----
 342
----
  4
```

b Multiplication by multiples of 10 should be thoroughly understood.

```
 436
x 20
----
8720
----
```

c Long multiplication should start with the left hand digit of the multiplier.

i

```
 436
x 24
----
8720
1744
----
10464
----
```

ii

```
 436
x204
----
87200
 0000
 1744
----
88944
----
```

The '0's row may be dispensed with as soon as convenient.

```
 436
x204
----
87200
 1744
----
88944
----
```

4 DIVISION

Note: i $\text{DIVISOR}\overline{)\text{DIVIDEND}}^{\text{QUOTIENT}}$

ii It is recommended that in both short and long division the quotient should be placed <u>above</u> the dividend with the digits in their appropriate positions.

iii The answer need not be re-written.

iv The use of a curved line $\overbrace{\quad}$ is recommended to avoid confusion with '1'.

v Any remainder should be written alongside the quotient, as in the examples which follow.

(a)
$$7\overline{)7498}\quad 1071 \text{ rem. } 1 \text{ (not } +1)$$

(b)
$$\begin{array}{r} 205 \\ 22\overline{)4513} \\ \underline{44} \\ 113 \\ \underline{110} \\ 3 \end{array} \quad \text{rem. } 3$$

FRACTIONS

Note: The fundamental ideas of fractions should be established through many practical experiences before any mathematical recording of fractions is carried out, e.g. folding and cutting to bring out the concept of 'equal parts'. Early work on fractions should lead to a thorough understanding of:

i the meaning, reading and writing of simple fractions.

ii the terms 'numerator' (number of parts) and 'denominator' (name of part)

iii The number of parts in the whole - hence the change from mixed number to improper fraction and vice-versa.

iv The equivalence of fractions - the value of a fraction remains the same if numerator and denominator are multiplied, or divided, by the same number, e.g.

$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{24}{32} = \ldots\ldots\ldots$$

$$\frac{8}{12} = \frac{4}{6} =$$

Which leads to reducing a fraction to its lowest terms, e.g.

$$\frac{32}{48} = \frac{8}{12} = \frac{2}{3}.$$

1 Addition and subtraction of fractions having the same denominator:

(a) $\frac{3}{5} + \frac{1}{5}$
$= \frac{4}{5}$

(b) $\frac{7}{8} - \frac{2}{8}$
$= \frac{5}{8}$

(c) $\frac{4}{5} + \frac{3}{5}$
$= \frac{7}{5}$
$= 1\frac{2}{5}$

(d) $1\frac{1}{2} - \frac{1}{2}$
$= 1$

(e) $1\frac{1}{4} - \frac{3}{4}$
$= \frac{5}{4} - \frac{3}{4}$
$= \frac{2}{4}$
$= \frac{1}{2}$

2 **Addition and subtraction of fractions having different denominators - making use of equivalent fractions.**

(a) $\frac{3}{4} + \frac{5}{8}$

$= \frac{6}{8} + \frac{5}{8}$

$= \frac{11}{8}$

$= 1\frac{3}{8}$

(b) $\frac{5}{6} + \frac{3}{4}$

$= \frac{10}{12} + \frac{9}{12}$

$= \frac{19}{12}$

$= 1\frac{7}{12}$

(c) $\frac{5}{6} - \frac{3}{4}$

$= \frac{10}{12} - \frac{9}{12}$

$= \frac{1}{12}$

3 **Dealing with mixed numbers:**

(a) $4\frac{2}{3} + 2\frac{3}{4}$

$= 4\frac{8}{12} + 2\frac{9}{12}$

$= 6\frac{17}{12}$

$= 7\frac{5}{12}$

(b) $4\frac{2}{3} - 2\frac{3}{4}$

$= 4\frac{8}{12} - 2\frac{9}{12}$

$= 3\frac{12 + 8}{12} - 2\frac{9}{12}$

$= 1\frac{11}{12}$

4 **Multiplication by a whole number:**

Note: **The extension of the meaning of 'multiply' should be introduced by considering the equivalence of '$\times$' and 'of'.**

(a) $\frac{2}{3} \times 6$

$= \frac{2}{3} \times \frac{6}{1}$

$= \frac{12}{3}$

(b) $\frac{1}{4} \times 6$

$= \frac{1}{4} \times \frac{6}{1}$

$= \frac{6}{4}$

$= \frac{3}{2}$

$= 1\frac{1}{2}$

$\frac{1}{4}$ of 6

$= \frac{1}{4} \times \frac{6}{1}$

.

.

.

.

.

.

$= 1\frac{1}{2}$

5 Multiplication by a fraction:

(a) $\frac{1}{4} \times \frac{1}{2}$

$= \frac{1}{8}$

(b) $\frac{2}{3} \times \frac{3}{4}$

$= \frac{6}{12}$

$= \frac{1}{2}$

6 Multiplication involving mixed numbers:

(a) $6 \times 2\frac{1}{4}$

$= \frac{6}{1} \times \frac{9}{4}$

$= \frac{54}{4}$

$= \frac{27}{2}$

$= 13\frac{1}{2}$

(b) $3\frac{1}{2} \times 2\frac{1}{4}$

$= \frac{7}{2} \times \frac{9}{4}$

$= \frac{63}{8}$

$= 7\frac{7}{8}$

7 Division by a whole number:

$\frac{2}{3} \div 6$

$= \frac{2}{3} \div \frac{6}{1}$

$= \frac{2}{3} \times \frac{1}{6}$

$= \frac{2}{18}$

$= \frac{1}{9}$

8 Division by a fraction:

$\frac{2}{3} \div \frac{5}{6}$

$= \frac{2}{3} \times \frac{6}{5}$

$= \frac{12}{15}$

$= \frac{4}{5}$

9 Division involving mixed numbers:

$= 2\frac{1}{2} \div 3\frac{1}{3}$

$= \frac{5}{2} \div \frac{10}{3}$

$= \frac{5}{2} \times \frac{3}{10}$

$= \frac{15}{20}$

$= \frac{3}{4}$

BEST COPY AVAILABLE

DECIMALS

A thorough understanding of the Decimal Notation is required. Stress the importance of place value again - extending to the right of the decimal point; introduce column headings "tenths", "hundredths" etc. Oral work is essential at this stage. Stress the importance of correctly naming Decimal Parts, e.g. 25.36, read as "twenty five point three six".

1 Addition and Subtraction (All arithmetic processes are as previously described).

Care should be taken in placing figures in correct columns. e.g. add together 3.65, 0.36, 0.05, 7.09:-

Arrange the decimal points IN LINE UNDER EACH OTHER, then the figures fall in the correct columns.

Set out as

$$\begin{array}{r} 3.65 \\ 0.36 \\ 0.05 \\ +7.09 \\ \hline \\ \hline \end{array}$$

Subtract:

(a) 15.362 from 21.8

It is essential to make columns regular by introducing 0's where necessary.

Set out as

$$\begin{array}{r} 21.800 \\ -15.362 \\ \hline \\ \hline \end{array}$$

(b) 15 - 3.7

Set out as

$$\begin{array}{r} 15.0 \\ -\ 3.7 \\ \hline \\ \hline \end{array}$$

2 Multiplication and Division by Powers of 10 should be thoroughly understood.

Examples

5.236	x	10	=	52.36
8.754	x	100	=	857.4
75.93	÷	10	=	7.593
987.6	÷	100	=	9.876
75	÷	10	=	7.5 (Decimal point is introduced).

3 **Multiplication of Two Decimals:**

2.5 x 3.5

Ignore Decimal Points and set out as

```
  25          Has 1 place of decimals for 2.5
x 35          Has 1 place of decimals for 3.5
----
 750
 125
----
 875          1 + 1 = 2 places of decimals in the
----                 answer.
```

2.5 x 3.5 = 8.75

Eventually 18.72 x 4.6

```
 18.72
x  4.6       Count the decimal places, but ignore
------       the decimal points
 74880
 11232
------
 86.112      3 places of decimals in answer
------
             Encourage writing of answer as
             Answer 86.112
```

4 **Adding "0's" as required:**

(a) 0.3 x 0.2

```
 0.3
x0.2
----
0.06
----
```

Answer must contain two places of decimals

Answer 0.06

(b) 0.321 x 0.048

```
 0.321
x0.048
------
 12840
  2568
------
0.015408
------
```

Answer must contain 6 places of decimals.

Answer 0.015408

5 **Division:**

Convert the divisor into a whole number by multiplication by powers of 10. Multiply the other number (dividend) by the same power of 10. Now treat as straight forward long division with the decimal point fixed.

(a) 3.95 ÷ 0.5

Becomes 39.5 ÷ 5 (by multiplication of both numbers by 10)

```
    7.9
  ------
5 ) 39.5
    35
    ---
     45
     45
     --
      0
```

Answer 7.9

(b) $37.395 \div 0.27$

Becomes $3739.5 \div 27$

$$
\begin{array}{r}
138.5 \\
27 \overline{)\,3739.5} \\
27 \\
\hline
103 \\
81 \\
\hline
229 \\
216 \\
\hline
135 \\
135 \\
\hline
0
\end{array}
$$

Answer 138.5

DECIMAL PRESENTATION OF METRIC UNITS

The foregoing examples of decimal recording are equally applicable when working in metric units. In each case the metric unit of measurement employed must be clearly defined by the use of the following standard abbreviations:

Length	Mass/Weight	Capacity
mm - millimetres	mg - milligrams	ml - millilitres
dm - decimetre	g - grams	l - litres
cm - centimetres	kg - kilograms	
m - metres		
km - kilometres		

Area	Volume
cm^2 - square centimetres	cm^3 - cubic centimetres
m^2 - square metres	m^3 - cubic metres

Examples of metric recording are as follows:

ADDITION

mm	cm	m	km	g	kg	l
19	52	2.25	3.255	152	3.625	2.250
23	31	3.46	4.116	246	4.825	3.265
+15	+14	+1.52	+3.729	+138	+1.455	+4.825

SUBTRACTION

mm	cm	m	km	g	kg	l
65	58	9.25	4.255	348	6.825	9.625
-24	-26	-1.36	-2.125	-212	-3.255	-2.257

MULTIPLICATION

mm	cm	m	km	g	kg	ℓ
25	36	2.56	1.625	136	1.225	2.625
x 3	x 2	x 3	x 4	x 3	x 3	x 4

DIVISION

$3\overline{)69}$ mm $5\overline{)95}$ cm $4\overline{)5.32}$ m $5\overline{)6.275}$ km $4\overline{)128}$ g $6\overline{)7.326}$ kg

$5\overline{)2.545}$ ℓ

It must be ensured that the answer is expressed in the units of the question.

MONEY

Note: i For amounts below £1.00 write the number of pence, followed by p, e.g.

17p 99p $3\frac{1}{2}$p

ii For amounts of £1.00 and above prefix by the £ sign but DO NOT add p at the end, e.g.

£1.07 £17.21 £11.03½

iii The 0 as a place-holder must be stressed, e.g.

£1.07 not £1.7
£15.03½ not £15.3½

iv The 'units' column of the pence amount must always be filled, e.g.

30p £1.30 £15.00½

v The 'penny point' (which separates £ from p) always goes between the second and third digits from the right.

vi If the answer is written separately then the £ or p sign should be added to it.

1 Addition (Vertical display):

(a) If all the addends are less than £1.00 then the column may be headed with either p or £, e.g.

p	p		£	£
3	27		0.03	0.27
+37	+95		+0.37	+0.95
40	122	or £1.22	0.40	1.22

(b) If ANY addend is written as £1.00 or more then the column should be headed by £, e.g.

£
0.27
+3.75
4.02

2 Subtraction (Vertical display) :

(a)

p
34
- 3
31

£
0.34
-0.03
0.31

(b) If the larger amount is £1.00 or more then the column should be headed by £, e.g.

£
3.75
-0.27
3.48

3 Multiplication :

£
3.48
x 4
13.92

4 Division :

$$6 \overline{)\,3.48} = 0.58$$

3.0
48
48
0

CONVERSIONS OF DECIMALS, FRACTIONS AND PERCENTAGES

The foregoing recommended methods for calculations involving fractions and decimals are applicable here.

1 Changing fractions to percentages :

Multiply fraction by 100

$\frac{3}{4}$
$= (\frac{3}{4} \times 100)\%$
$= 75\%$

$\frac{1}{3}$
$= (\frac{1}{3} \times 100)\%$
$= 33\frac{1}{3}\%$

2 Changing decimals to percentages :

Multiply decimal by 100

0.75
$= (0.75 \times 100)\%$
$= 75\%$

0.33
$= (0.33 \times 100)\%$
$= 33\%$

3 **Changing percentages to fractions :**

Write as a fraction of 100 and reduce to lowest terms

54%	$32\frac{1}{2}\%$	120%
$= \frac{54}{100}$	$= \frac{32\frac{1}{2}\%}{100}$	$= \frac{120}{100}$
$= \frac{27}{50}$	$= \frac{65}{200}$	$= \frac{6}{5}$
	$= \frac{13}{40}$	$1\frac{1}{5}$

4 **Changing decimals to fractions :**

Write as fraction and reduce to lowest terms

0.54	0.325	1.2
$= \frac{54}{100}$	$= \frac{325}{1000}$	$= 1\frac{2}{10}$
$= \frac{27}{50}$	$= \frac{13}{40}$	$= 1\frac{1}{5}$

5 **Changing fractions to decimals :**

Divide numerator by denominator incorporating the decimal point

$\frac{3}{4}$ gives $4\overline{)3.00}$ $\quad 0.75$ $\quad \frac{3}{4} = 0.75$

$\frac{5}{9}$ gives $9\overline{)5.00}$ $\quad 0.55$ $\quad \frac{5}{9} = 0.\dot{5}$

$\frac{3}{7}$ gives $7\overline{)3.0000000}$ $\quad 0.4285714$ $\quad \frac{3}{7} = 0.\dot{4}28571\dot{4}$

Note: Dots above figures or enclosing groups of figures denote recurring part(s).

6 **Changing percentages to decimals :**

Divide by 100

$54\% = 0.54$

$30\% = 0.3$

$33\frac{1}{3}\% = 0.\dot{3}3\dot{3}$

Children should be encouraged to memorise simple conversions such as halves, quarters and tenths.